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In-medium chiral perturbation theory

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Abstract. We report on how to tackle the problem of establishing a chiral effective field theory in nuclear matter with explicit pion fields and in the presence of external sources (Ann. Phys. **297**, 27 (2002)). We have made use of the results of J.A. Oller (Phys. Rev. C **65**, 025204 (2002)) where the generating functional for the in-medium chiral $SU(2) \times SU(2)$ Lagrangian has been derived. Within this approach we develop the so-called standard power counting rules for the calculation of in-medium pion properties if the residual nucleon energies are of the order of the pion mass. In addition, for the case of vanishing residual nucleon energies, a modified scheme (non-standard counting) is introduced. For both schemes the pertinent scales where the chiral expansions have to break down are established as well. We have performed a systematic analysis of n-point in-medium Green functions up to and including next-to-leading order when the standard rules apply. These include the in-medium contributions to quark condensates, pion propagators, pion masses and couplings of the axial-vector, vector and pseudoscalar currents to pions.

PACS. 12.39.Fe Chiral Lagrangians – 11.30.Rd Chiral symmetries – 21.65.+f Nuclear matter

1 Introduction

In this paper we will report about a *systematic* study of the properties of pions and external sources in nuclear matter [1,2]. For typical nuclear densities, the interactions of the constituents of nuclear matter belong to the low-energy regime of strong interactions, such that one has to deal with phenomena in the non-perturbative sector of the underlying theory, QCD. Thus approximations are unavoidable. However, they should be controllable as in the methodology of effective field theories. Consequently, the aim is to create an in-medium QCD effective field theory that allows to estimate the errors when the pertinent perturbative expansion is truncated at some order.

The low-energy effective field theory of QCD in the vacuum is chiral perturbation theory (ChPT) [3,4]. It is believed that the approximate $SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$ chiral symmetry of QCD is spontaneously broken down to its vectorial subgroup, $SU(2)_{\mathrm{L+R}}$, with the appearance of three (Pseudo-)Goldstone bosons which can be identified with the three pion states, $\pi^{\pm}, \pi^0$. The unique order parameter signaling this symmetry violation is the finiteness of the square of the weak pion decay constant in the chiral limit, $f^2 \neq 0$; in fact, $f \simeq f_\pi = 92.4\,\mathrm{MeV}$. In addition, the chiral symmetry is also explicitly broken because the current up and down quarks have a small mass (small compared to a typical hadronic scale of 1 GeV). Due to Goldstone's theorem, the interactions of the pions with themselves or matter fields must vanish as the three-momentum and energy go to zero in the chiral limit. This in turn allows for a systematic treatment of such processes in the framework of an effective field theory, namely ChPT [3,4], in terms of a simultaneous expansion in external momenta and mass terms.

ChPT allows not only to tackle processes involving pions, but as well to consider nucleons (baryons). These massive states are included as matter fields chirally coupled to pions and external sources.

There are many articles [5–8] in the literature where ChPT Lagrangians, at most bilinear in the nucleon fields, are applied to the nuclear matter case in the mean-field approach, *i.e.* where the information contained in the vacuum ChPT Lagrangians is kept, but the baryon propagators in the medium are replaced by local densities in the mean-field approach. Proceeding in this way one looses the chiral counting in the medium, since the non-local nucleon correlations, due to the baryon propagators, are not considered. In fact, such contributions can be of the same or even of lower chiral order as those terms kept in the mean-field approach. Especially, they are the dominant contributions when the energy flowing through the baryon propagator is of the order of a nucleonic kinetic energy [1].

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2 In-medium generating functional

Our starting point is the ChPT Lagrangian supplemented by external fields [4]. As in the mean-field approaches, multi-nucleon interactions will be left out here. Thus only the perturbative expansion of the chiral lagrangians $\mathcal{L}_{\pi\pi}$ and $\mathcal{L}_{\overline{\psi}\psi}$ (the latter is bilinear in the nucleon field) in powers of the external four-momenta of the pions, the external three-momenta of the nucleons and the quark masses will be taken into account. We apply the generating functional techniques of ref. [2]; *i.e.* the *in-medium* $SU(2)\times SU(2)$ generating functional is written as a path integral over the chiral field $U\equiv uu\equiv\exp[i\phi/f]$, where the ground states at asymptotic times have been replaced by the Fermi sea states of protons and neutrons and where secondly the nucleon fields have been integrated out:

$$\begin{aligned}
e^{i\mathcal{Z}[v,a,s,p]} &= \int[\mathrm{d}U] \\
&\times\exp\Biggl\{ i\int \mathrm{d}x\,\mathcal{L}_{\pi\pi} + \int\frac{\mathrm{d}\boldsymbol{p}}{2E(p)(2\pi)^3}\int \mathrm{d}x\mathrm{d}y \\
&\times e^{ip(x-y)}\mathrm{Tr}\left[-iA\bigl(I_4-D_0^{-1}A\bigr)^{-1}|_{(x,y)}(\not{p}+m_N)n(p)\right] \\
&-\frac{1}{2}\int\frac{\mathrm{d}\boldsymbol{p}}{2E(p)(2\pi)^3}\int\frac{\mathrm{d}\boldsymbol{q}}{2E(q)(2\pi)^3}\int \mathrm{d}x\mathrm{d}x'\mathrm{d}y\mathrm{d}y'\,e^{ip(x-y)} \\
&\times e^{-iq(x'-y')}\mathrm{Tr}\Bigl[-iA\left(I_4-D_0^{-1}A\right)^{-1}|_{(x,x')}(\not{q}+m_N)n(q) \\
&\times(-iA)\left(I_4-D_0^{-1}A\right)^{-1}|_{(y',y)}(\not{p}+m_N)n(p)\Bigr]+\cdots\Biggr\} \\
&\equiv\int[\mathrm{d}U]\,\exp\left\{ i\int \mathrm{d}x\widetilde{\mathcal{L}}_{\pi\pi}[U;v,a,s,p]\right\}. \qquad (1)
\end{aligned}$$

Here $A=D_0(x)-D(x)$ is the difference between the free Dirac operator $D_0=i\gamma^\mu\partial_\mu-m_N$ and the Dirac operator $D(x)$ of $\mathcal{L}_{\overline{\psi}\psi}\equiv\overline{\psi}(x)D(x)\psi(x)$, while I_4 is the unit operator in 4 dimensions. Thus $A(x)$ is a vertex operator in terms of pion legs and external (vector v, axial-vector a, scalar s, and pseudoscalar p) sources. The diagonal flavor matrix $n(p)=\mathrm{diag}[\theta(k_\mathrm{F}^{(p)}-|\boldsymbol{p}\,|),\theta(k_\mathrm{F}^{(n)}-|\boldsymbol{p}\,|)]$ parametrizes the upper cutoff for the three-momentum integrations, where $k_\mathrm{F}^{(p)}$ and $k_\mathrm{F}^{(n)}$ are the proton and neutron Fermi momenta, respectively. Finally, $E(p)=(\boldsymbol{p}^{\,2}+m_N^2)^{1/2}$ is the on-shell energy of the nucleon of mass m_N.

In this way the *in-medium* ChPT Lagrangian $\widetilde{\mathcal{L}}_{\pi\pi}$, is derived. The structure is the same as for vacuum ChPT, except for the important difference that the resulting Lagrangian is *non-covariant* as well as *non-local.* Especially, there appears now a non-local vacuum vertex $\Gamma\equiv-iA(I_4-D_0^{-1}A)^{-1}$ that generates a geometric series in terms of the local interaction operator A and the free Dirac propagator D_0^{-1}. The vertex-operator A in turn is subject to a chiral power expansion. The pertinent interaction operators of first and second order read

$$\begin{aligned}
A^{(1)} &= -i\gamma^\mu\Gamma_\mu-ig_A^0\gamma^\mu\gamma_5\Delta_\mu\ , \\
A^{(2)} &= -c_1\langle\,\chi_+\,\rangle-2c_2\langle\Delta_\mu\Delta_\nu\rangle\frac{D^\mu}{m_N}\frac{D^\nu}{m_N} \\
&\quad+2c_3\langle\Delta_\mu\Delta^\mu\rangle-c_5\left(\chi_+{-}\tfrac{1}{2}\langle\chi_+\rangle\right)+\cdots\ ,
\end{aligned}$$

$$\text{with}\ \ \chi_+\equiv u^\dagger\chi u^\dagger+u\chi^\dagger u\ ,\quad \chi\equiv-2\frac{\langle\bar{q}q\rangle}{f^2}(s+ip)\ ,$$

$$2\begin{Bmatrix}\Gamma_\mu\\ \Delta_\mu\end{Bmatrix}\equiv\left[u^\dagger,\partial_\mu u\right]_\mp-iu^\dagger(v_\mu{+}a_\mu)u\pm iu(v_\mu{-}a_\mu)u^\dagger.$$

Here $D_\mu\equiv\partial_\mu+\Gamma_\mu$ and $\langle\cdots\rangle$ is defined as the trace in the flavor space. The c_i's are low-energy constants of $\mathcal{O}\left(p^2\right)$ terms and therefore finite, see ref. [9,10] for more details. $A^{(1)}$ includes the S-wave Weinberg-Tomozawa term and the derivative P-wave pion-nucleon coupling, while the sigma-term (proportional to c_1) and the so-called range terms (proportional to c_2 and c_3) appear in $A^{(2)}$. The generalized *in-medium* vertices consist of non-local vacuum vertices Γ connected through the exchange of on-shell Fermi-sea states with pertinent three-momenta smaller than $k_\mathrm{F}^{(p)}(k_\mathrm{F}^{(n)})$. These generalized vertices may still be linked to each other by pion legs from the local Lagrangian $\mathcal{L}_{\pi\pi}$. The lowest order *in-medium* vertices contain only $A^{(1)}$ operators; the next-to-leading order is obtained when one $A^{(2)}$ operator is involved. We refer to ref. [1] for the pertinent Feynman rules for the computation of connected in-medium graphs in momentum space.

3 Results

The novel results that we have obtained in ref. [1] can be summarized as follows:

i) In contrast to most previous works, which adopt the mean-field approach or many-body calculations, the in-medium chiral counting has been established including contributions from baryon propagators. The counting scheme depends on the energy flowing into the nucleon lines. This leads to a separate treatment of the standard and the non-standard case (*i.e.*, vanishing energy flow), respectively. In the standard case, the chiral expansion of pion properties in the medium starts with terms at $\mathcal{O}(p^4)$, and the next-to-leading order corrections appear at $\mathcal{O}(p^5)$, in the non-standard case (*e.g.*, in the in-medium $\pi\pi$ scattering) these orders are $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$, respectively, whereas the in-vacuum power counting starts at $\mathcal{O}(p^2)$ with next-to-leading corrections of $\mathcal{O}(p^4)$.

ii) In the vacuum, the scale at which the chiral expansion is bound to break down is approximately $\Lambda_\chi\simeq 1\,\mathrm{GeV}\sim 4\pi f_\pi$. In the medium, there appear two further scales. These are $\sqrt{6}\pi f_\pi\simeq 0.7$ GeV and $6\pi^2 f_\pi^2/2m_N\simeq 0.27$ GeV for the standard or the non-standard counting rules, respectively. If there are P-wave interactions, the two in-medium scales have to be reduced by a factor of $1/g_A$ or $1/g_A^2$, respectively.

iii) We have re-derived, from the effective field theory point of view, the in-medium quark condensates in symmetric nuclear matter [12,13] and have further extended them to the non-symmetric case:

$$\langle\Omega|\bar{q}q|\Omega\rangle = \langle\bar{q}q\rangle_{\rm vac}\left[1-\frac{2\sigma}{f^2M_\pi^2}\widehat{\rho}\pm\frac{4c_5}{f^2}\overline{\rho}\right].$$

Here the $\pm$ sign applies for the $\langle\bar{u}u\rangle$ and $\langle\bar{d}d\rangle$ condensate, respectively. $|\Omega\rangle$ is the nuclear matter background, while $\widehat{\rho}\equiv\frac{1}{2}(\rho_p+\rho_n)$ and $\overline{\rho}\equiv\frac{1}{2}(\rho_p-\rho_n)$ are defined as the isospin symmetric and asymmetric combinations of the proton and neutron density, ρ_p and ρ_n respectively. The quantity $\sigma=-4c_1M_\pi^2\approx 44\pm 8\pm 7\,$MeV is pion-nucleon sigma term [11,14] and M_π is the vacuum pion mass. The low-energy constant $c_5 = -(0.09\pm 0.01)\,{\rm GeV}^{-1}$ parametrizes the strong isospin breaking [14]. It is very suppressed as compared to $2c_1 = 2(-0.81\pm 0.12)$ ${\rm GeV}^{-1}$ [15]. Hence the presence of a c_5 term does not induce a sizable change of the quark condensates at finite density, as studied in refs. [7,12,13] for the symmetric nuclear matter case. At normal nuclear matter density $\rho=\rho_0$, there is a 35% reduction of the condensates. This is compatible with a partial restoration of chiral symmetry.

iv) We have obtained the inverse pion propagator up to $\mathcal{O}(p^5)$. The corresponding dispersion relation between the energy ω and the three-momentum $\boldsymbol{q}$ of an in-medium pion π^a $(a=+,-,0)$ is given by

$$\begin{aligned}\omega^2 - M_{\pi^a}^2\left(1+c_1\frac{8\widehat{\rho}}{f^2}\right)+\frac{4\widehat{\rho}}{f^2}\omega^2\left(c_2+c_3-\frac{g_A^2}{8m_N}\right)\\ -\boldsymbol{q}^2\left(1+\frac{4\widehat{\rho}}{f^2}c_3-\frac{g_A^2\widehat{\rho}}{m_Nf^2}\right)-\frac{g_A^2\widehat{\rho}}{2f^2m_N}\frac{\left(\boldsymbol{q}^2\right)^2}{\omega^2}\\ -\delta_{a0}\frac{4M_{\pi^a}^2\overline{\rho}}{f^2}\frac{m_u-m_d}{m_u+m_d}c_5\pm\delta_{a\pm}\left(\frac{g_A^2\boldsymbol{q}^2}{f^2\omega}-\frac{\omega}{f^2}\right)\overline{\rho}=0.\end{aligned} \quad (2)$$

Here M_{π^a} is the vacuum mass of pion π^a at $\mathcal{O}(p^4)$. For symmetric matter $(\overline{\rho}=0)$ and in the chiral limit $(m_q=0)$ the dispersion law simplifies to $\omega^2=\boldsymbol{q}^2\left(1-4\widehat{\rho}c_2/f^2\right)$. Thus the in-medium pion velocity $\widetilde{v}={\rm d}\omega/{\rm d}|\boldsymbol{q}|=1-2\widehat{\rho}c_2/f^2$ is less than the vacuum speed of light $(c=1)$ for the empirical value of the low-energy constant $c_2 = (3.2\pm 0.25)\,{\rm GeV}^{-1}$ of ref. [9,10]. Inserting furthermore the value $c_3=(-4.70\pm 1.16)\,{\rm GeV}^{-1}$ from ref. [15], we have established from eq. (2) that chiral symmetry can account for the observed shift of the mass of the negative pion in deeply bound pionic states in ^{207}Pb [16,17], see also [18].

v) The wave function renormalizations of the in-medium pion fields, $\langle\Omega|\pi^\alpha(x)|\pi^\beta(p)\rangle\equiv Z_\alpha(\boldsymbol{q}^2)^{-1/2}\delta_{\alpha\beta}e^{-ipx}$, have been established from general principles via the equal-time commutation relations.

vi) With the help of the generating-functional formalism we can also study the coupling of pions with axial-vector and pseudoscalar sources [1]. In particular, we have shown that there is a splitting of the temporal and space-like component of the in-medium pion decays constants f_t and f_s, respectively [19,7,8]), which we have determined in the isospin-limit $(\overline{\rho}=m_u-m_d=0)$ and at threshold as

$$f_{t/s}=f_\pi\left\{1\pm\frac{2\widehat{\rho}}{f^2}\left(c_2\pm c_3\mp\frac{g_A^2}{8m_N}\right)\right\}. \quad (3)$$

Moreover, we have confirmed that in-medium corrections up to $\mathcal{O}(p^5)$ do not spoil the validity of the Gell-Mann-Oakes-Renner relation, see ref. [7]. The decrease with increasing density for both, the quark condensates and the temporal component of the pion decay constant f_t seem to indicate a partial chiral symmetry restoration with increasing density. In addition, we have checked that the QCD Ward identities between the axial-vector currents and pseudoscalar (isovector and isoscalar) currents hold to $\mathcal{O}(p^5)$.

vii) Finally, the in-medium $\pi\pi$ scattering has been studied up to $\mathcal{O}(p^3)$ since in this case the non-standard counting occurs. Note that not only the in-medium corrections start at a lower order than in the standard case, namely already at $\mathcal{O}(p^3)$, but also the scale, below which the perturbative expansion is applicable, decreases. This leads to a rapid increase of the in-medium corrections with density. In fact, already at $k_{\rm F}\simeq 200$ MeV, or at a density of just $\sim 0.4\rho_0$, they are of the same size as the lowest order ChPT results.

4 Outlook

There are still open problems, *e.g.* the inclusion of multi-nucleon contact interactions remains a challenge, especially since the S-wave nucleon-nucleon interactions are enhanced because of the largeness of the S-wave scattering lengths related to the presence of shallow NN bound states. Moreover, this task has to be combined with simultaneous pion-loop calculation in order to guarantee that *all* in-medium $\mathcal{O}(p^6)$ contributions are taken into account. Finally, a search for a non-perturbative scheme that would allow to recover the scale $\sqrt{6}\pi f_\pi$, even in the case of the non-standard counting or even in the presence of multi-nucleon contact interactions, should be high on the list of future investigations in this field.

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